DETERMINATION OF DUAL-MODE Q FACTORS FROM MEASURED DATA

W. Perry Wheless, Jr.* and Darko Kajfez**

*ECE Dept., New Mexico State University, Las Cruces, NM 88003
**EE Dept., University of Mississippi, University, MS 38677

ABSTRACT

An iterative least squares parameter estimation procedure is applied to large sets of measured data, obtained with the aid of a vector network analyzer, to determine the unloaded Q factors of two resonant modes in near frequency proximity. The procedure is capable of analyzing the data of very closely spaced dual resonances, such as typically occur in dual-mode filters utilizing degenerate hybrid modes in dielectric resonators.

INTRODUCTION

This study begins with the results of [1] and makes the extensions (a) from isolated single resonances to the more complex case of two resonances occurring in close frequency proximity, and (b) from reflection-type measurement data to the analysis of reaction-type (e.g., MIC environment) measurement data. The dual resonance lumped-element equivalent circuit model is shown in Figure 1. Input impedance, Z, for this circuit with resonant frequencies ω_1 and ω_2 is

$$Z = jX_e + j2R_cQ_e\delta_r + \frac{\kappa_1 R_c}{1+jQ_1\Omega_1} + \frac{\kappa_2 R_c}{1+jQ_2\Omega_2} \quad (1)$$

where

κ_1 = coupling coefficient for mode 1

κ_2 = coupling coefficient for mode 2

Q_1 = unloaded Q factor for mode 1

Q_2 = unloaded Q factor for mode 2

ω_r = reference frequency = $\frac{\omega_1 + \omega_2}{2}$

$$\delta_r = \frac{\omega - \omega_r}{\omega_r}$$

$$\Omega_1 = \frac{\omega}{\omega_1} - \frac{\omega_1}{\omega}$$

and

$$\Omega_2 = \frac{\omega}{\omega_2} - \frac{\omega_2}{\omega}$$

Constants X_e and Q_e are considered in [2]. The circuit model is thus described by eight real constants: X_e, Q_e, ω_1, ω_2, κ_1, κ_2, Q_1, and Q_2.

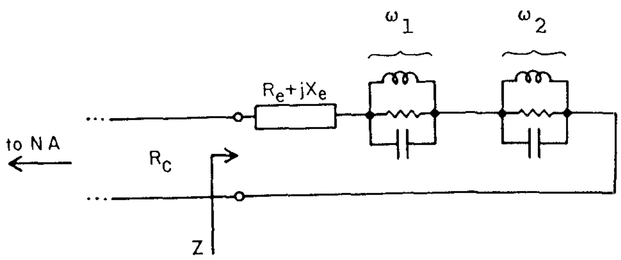

Figure 1. Dual resonance lumped-element equivalent circuit.

The primary objective of this particular study is to discern, from measured data, the unloaded Q factors and coupling coefficients associated with the individual resonances. As measurements are performed on a dielectric resonator placed in an actual operating environment, the resulting values of the unloaded Q factors incorporate both the conductor losses and the dielectric losses of the resonator and its surroundings. Knowledge of these parameter values is of great interest to the microwave filter designer.

This material is based upon work supported by the National Science Foundation under Grant ECS-8443558.

0149-645X/87/0000-0375$01.00 © 1987 IEEE

IMPEDANCE DE-EMBEDDING

The desired impedance, Z, may be de-embedded from a test fixture of the type illustrated in Figure 2, under the assumptions that the dielectric resonator is placed in the fixture mid-way between the two ports and that the test fixture construction is symmetrical with respect to the center plane (O-O' in Figure 2a). The de-embedded impedance at a given frequency is approximated by

$$Z \approx 2Z_{01}\frac{\left[\frac{S_{21e}}{S_{21o}} - 1\right](1 + j2x)}{1 + j2x - 2(\delta+jx)e^{-2\gamma\ell}} \qquad (2)$$

where

Z_{01} = effective characteristic impedance of the microstrip line, from tdr measurements

S_{21e} = average of measured S_{12} and S_{21} parameters, with the test fixture empty (i.e., with the dielectric resonator removed)

S_{21o} = measured S_{21} with the dielectric resonator in place in the test fixture

$\gamma\ell = \alpha\ell + j\beta\ell$. Microstrip half-length ℓ is physically measured.

The phase shift $\beta\ell$ can be calculated from the physical dimensions of the microstrip section. The attenuation $\alpha\ell$ is found from

$$2\alpha\ell \approx \ln\left[\frac{\left|\frac{1}{S_{21e}}\right|}{\sqrt{1+4x^2}}\right]. \qquad (3)$$

In (2) and (3),

$$x \approx \frac{1}{\cos\beta\ell}\left[\frac{1}{2}\mathrm{Im}\left[\frac{S_{11e}}{S_{21e}}\right] + \delta \sin\beta\ell\right], \qquad (4)$$

where S_{11e} is the average of measured S_{11} and S_{22} parameters with the test fixture empty.

In (2) and (4), $\delta = \dfrac{Z_0 - Z_{01}}{2\sqrt{Z_0 Z_{01}}}$

where Z_0 is the system characteristic impedance (50 Ω).

LEAST SQUARES ESTIMATION

The circuit model function can be fit to the measured data using the same least squares procedure as [1], but with eight total parameters in the dual resonance case. It has been concluded [3] that weighting has no appreciable effect in this class of resonance modeling problems, and an unweighted least squares analysis is conducted in all example cases which follow.

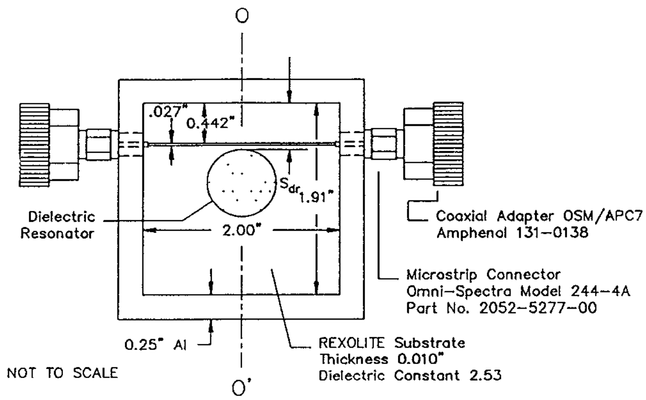

Figure 2a. Reaction type measurements test fixture, top view.

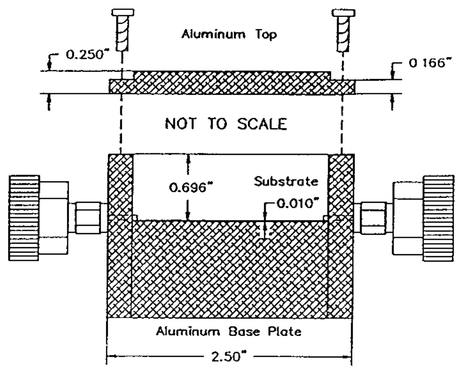

Figure 2b. Reaction type measurements test fixture, cross-sectional view.

The same initial parameter estimation computer implementation used for isolated single resonances [1] is applicable to the dual resonance case as well, since the data are the same impedance versus frequency information.

EXPERIMENTAL RESULTS

In an actual case study, using S-parameter data collected with a vector network analyzer system, a Murata Erie model DRD178UC079B dielectric resonator was employed. To study a variable dual resonance condition, a test fixture top plate is used which has a protruding coupling screw of adjustable penetration (P_s). The top plate is shown in Figure 3.

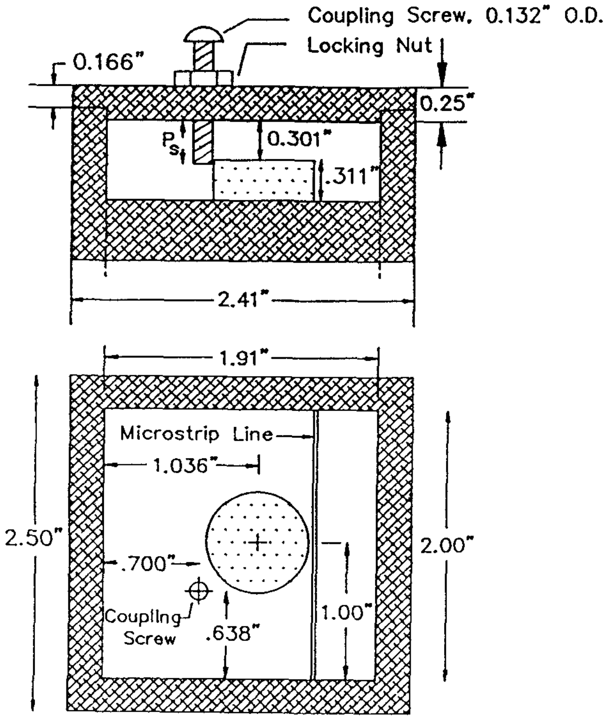

Figure 3. Test fixture for dual mode measurements.

The selected HEM mode responds to increasing screw penetration into the test fixture enclosure by progressing from undercritically coupled orthogonal modes (with the screw retracted), through critical coupling, and on into various levels of overcoupled response.

With a coupling screw penetration of P_s = 7.14 mm, the state is one of strongly overcoupled response. The analysis for this case is summarized in Table I, and the model fit against the data points after ten iterations is shown in Figure 4.

When the coupling screw penetration is reduced to P_s = 5.56 mm, a less overcoupled condition is achieved. The analysis for this circumstance is given in Table II, and the model fit after seven iterations is shown in Figure 5.

With the screw further retracted to P_s = 3.97 mm, a near critical coupling state is realized. Table III summarizes this case, and the model fit after two iterations is given in Figure 6.

The resonant frequencies in this final illustrative example differ by only 0.54 MHz out of approximately 5 GHz. Thus, the procedure is capable of analyzing the data of very closely spaced dual resonances.

TABLE I
Solution Summary for Dual Resonance, Strongly Overcoupled

	Initial Estimates	Iteration 10 Values ±1 Standard Error
$X_e(\Omega)$	0.0	1.134±0.136
$Q_e(\Omega)$	0.0	1.811±2.952
Q_1	5000	5305±178
Q_2	5000	3944±137
f_1(GHz)	5.2092	5.20988 ± 10^{-5}
f_2(GHz)	5.2123	$5.21275\pm1.4(10^{-5})$
κ_1	0.33	0.354±0.007
κ_2	0.33	0.291±0.006

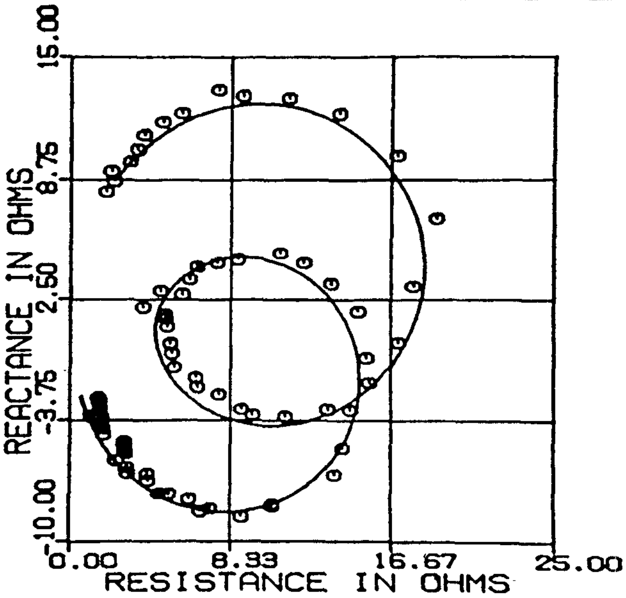

Figure 4. Model fit to strongly overcoupled dual resonance data after ten iterations.

CONCLUSION

A measurement procedure has been described, suitable for an accurate determination of equivalent circuits for single and multiple modes in microwave resonators. The numerical data processing technique is applicable to reflection type measurements as well as for reaction type measurements of unloaded Q factors. Reaction type measurements for the dielectric resonator near a microstrip line are enhanced by de-embedding the appropriate impedance data from measured values of S_{11} and S_{12}, obtained with an a

TABLE II
Solution Summary for Dual Resonance, Overcoupled

	Initial Estimates	Iteration 7 Values ±1 Standard Error
$X_e(\Omega)$	0.0	1.146±0.115
$Q_e(\Omega)$	0.0	6.103±2.949
Q_1	5000	6520±272
Q_2	5000	3650±144
f_1(GHz)	5.2108	5.21149±9(10^{-6})
f_2(GHz)	5.2125	5.21276±2.0(10^{-5})
κ_1	0.50	0.411±0.009
κ_2	0.50	0.326±0.007

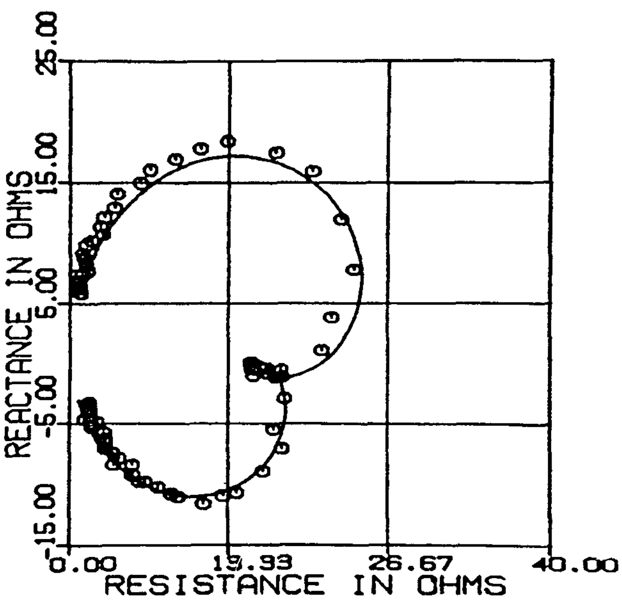

Figure 5. Model fit to overcoupled dual resonance data after seven iterations.

automated network analyzer system. The de-embedding procedure takes into account the fact that the characteristic impedance of the microstrip may differ slightly from the nominal value of 50 Ω, that the microstrip line has an attenuation, and that the transition from microstrip to coaxial line may be represented by a lumped reactance.

TABLE III
Solution Summary for Dual Resonance, Near Critical Coupling.

	Initial Estimates	Iteration 2 Values ±1 Standard Error
$X_e(\Omega)$	0.0	0.813±0.112
$Q_e(\Omega)$	0.0	1.802±2.931
Q_1	8000	7743±934
Q_2	4250	4506±154
f_1(GHz)	5.2122	5.21222±1.4(10^{-5})
f_2(GHz)	5.2127	5.21276±3.3(10^{-5})
κ_1	0.25	0.394±0.034
κ_2	0.50	0.436±0.028

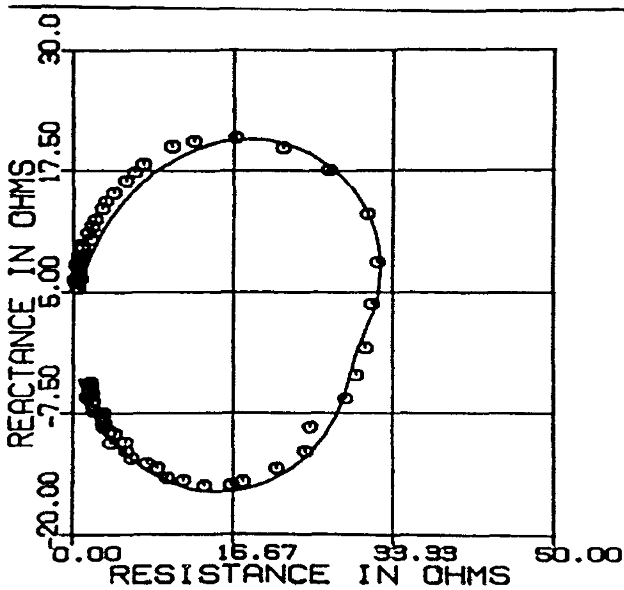

Figure 6. Model fit to dual resonance data after two iterations.

REFERENCES

1. W.P. Wheless, Jr. and D. Kajfez, "Microwave resonator circuit model from measured data fitting," in 1986 *IEEE MTT-S Int. Microwave Symp. Dig.*, pp. 681-684.
2. D. Kajfez and E. Hwan, "Q-factor measurement with network analyzer," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-32, no. 7, pp. 666-670, July 1984.
3. W.P. Wheless, Jr., "High-Precision Methods for the Experimental Characterization of Dielectric Resonators," Ph.D. Dissertation, University of Mississippi, August 1986 (available from University Microfilms, Ann Arbor, Publication No. 87-03481).